ABSTRACT

of dissertation for the Philosophy Doctor (PhD) degree in specialty “6D060300-Mechanics”

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INVESTIGATION OF THE TRANSLATIONAL-ROTATIONAL MOTION OF A THREE-AXIS NON-STATIONARY BODY IN THE FIELD OF ATTRACTION OF A NON-STATIONARY SPHERICAL BODY

General description of work

Classical celestial mechanics considers space bodies as material points. The analytical mechanics of a system of mutually gravitating free material points based on Newton's law is well-developed and remains relevant to this day. Its distinguishing feature is the consideration of translational-rotational motion of bodies with mutual interactions. However, even classical scholars of celestial mechanics (such as I. Newton, L. Euler, J. Lagrange, and others) acknowledged the limitations of the model of multiple material points, noting that modeling celestial bodies as material points lacks an adequate mathematical model for certain physical problems related to their dynamical evolution.

Modern observational astronomy confirms that celestial bodies are not only non-spherical but also cannot be treated as absolutely rigid bodies. This issue also applies to the well-known classical two-body problem. In the classical formulation, the problem is integrable when dealing with two point masses or two finite-size bodies with spherically symmetric mass distribution, and its general solution is well known.

Many Russian scientists—Y.V. Barkin, V.V. Beletsky, V.V. Vidyakin, N.I. Gamarnik, V.G. Demin, G.N. Duboshin, S.G. Zhuravlev, V.T. Kondurar, G.F. Osipova, T.K. Shinkarik—and foreign researchers—B. Gurta, P.B. Johnson, T.R. Kane, H. Kinoshita, P.K. Mishra, M. Pascal, and others—have contributed to solving the general problem of translational-rotational motion of rigid bodies. Kazakh scientists such as Zh.S. Erzhanov, A.A. Kalybaev, A.A. Baymukhametov and their students have extensively studied translational and rotational theories of celestial body motion.

Until the end of the 19th century, celestial mechanics dealt primarily with stationary celestial bodies. The differential equation describing the relative motion of two bodies with variable mass was first applied by Gilden in 1884. Significant results in the study of non-stationary space body motion were later achieved by V.G. Fesenkov, G.M. Idlis, T.B. Omarov, J.D. Hadjidemetriou, L.G. Lukyanov, E.N. Polyakhova, A.A. Bekov, A. Deprit, L. Floria, and others.

If at least one of the bodies is not spherically symmetric, then their mutual gravitational interaction depends on their geometric shapes and mass distributions, significantly complicating the problem. Over time, masses, sizes, shapes, and internal mass distributions change. This is supported by the stellar internal evolution theory, which shows that at different stages of development, stars experience compression or expansion. For example, during the red giant phase, a star's radius can increase 10–100 times depending on its mass. In close binary systems, changes in shape, size, mass transfer, and dissipation processes occur intensively. As a result, their mutual gravitational interaction becomes time-dependent, and the Newtonian potential explicitly varies with time. These factors significantly influence their dynamical evolution.

T.B. Omarov studied the dynamics of wide binary stars with intense corpuscular radiation and solved the problem of two bodies moving in a dust cloud. In parallel, Professor J. Hadjidemetriou conducted similar studies in Greece. The results, obtained independently, are known in dynamic astronomy as the "Omarov–Hadjidemetriou elements." These studies have deepened our understanding of the dynamical evolution of galactic, stellar, and planetary systems.

Russian scientist M.Yu. Barkin studied the perturbations in the rotational motion of celestial bodies with changing mass geometry, including Earth. Using Andoyer and action-angle variables, he proposed a method for accounting for seasonal and secular variations in geopotential coefficients.

M.Dzh. Minglibaev proposed a new intermediate motion model and canonical perturbation theory for non-stationary gravitational systems to study the motion of a point mass around a non-stationary spheroidal body with variable size and constant mass. A.A. Bekov proposed a general intermediate motion model for variable mass, size, and shape cases.

Actuality of the theme.

The study of translational-rotational motion of two mutually gravitating bodies with variable mass and size is one of the current and under-researched topics. It involves analyzing the translational motion of the mass center and the rotational motion around the mass center of natural and artificial non-stationary celestial bodies.

The main goal of work – to determine the effects of the variability in mass and dimensions of a triaxial body on the evolution of its translational-rotational motion in the gravitational field of a non-stationary spherical body.

Research tasks

1. To consider the physical formulation of the problem in binary gravitational systems, define celestial mechanics assumptions, and construct a mathematical model of the studied object.
2. To derive equations of motion of a non-stationary triaxial body in the gravitational field of a non-stationary spherical body using various approximations of the potential function and different systems of osculating elements. To expand the disturbing function.
3. To derive and analyze the evolutionary equations. To obtain approximate analytical solutions and compare them with numerical results.
4. To analyze the dynamic effects of the variability in mass and dimensions

of triaxial body on orbital and rotational motion around the mass center. To visualize the solutions.

The object of the research

The translational-rotational motion of a triaxial non-stationary body in the gravitational field of a non-stationary spherical body.

Research methods.

The study uses the Hamilton–Jacobi method, various variable transformations, different forms of perturbation theory including canonical perturbation theory, the mathematical apparatus of Hamiltonian systems of differential equations, analytical geometry of systems of material points, Gauss averaging method, and numerical methods of the Mathematica computer algebra system.

Scientific novelty of the dissertation results.

This dissertation derives perturbed motion equations for a triaxial body's translational-rotational motion using known methods of canonical perturbation theory.

The motion of a triaxial non-stationary body is considered using an analogue of the Delone–Andoyer variables, and secular perturbation equations are obtained.

Analytical analysis is performed on these equations, and numerical solutions of the perturbed differential equations are found. The solutions can serve as initial approximations in the study of complex problems in celestial mechanics.

The main provisions for the defense.

1. Perturbed motion equations for translational-rotational motion of a triaxial non-stationary body were derived.
2. The motion was considered in the analogue of Delone–Andoyer elements, and the corresponding evolutionary equations were obtained.
3. Analytical analysis was conducted on the evolutionary perturbation equations. The solutions were obtained numerically and visualized using computer algebra.

The reliability and validity of the results.

The scientific results are confirmed by correspondence with analogous real-world models and are consistent with previously obtained results in the case of constant mass and size, validating the theoretical model.

Theoretical and practical importance of the dissertation.

The results obtained in this dissertation define a new stage in the study of non-stationary space systems and serve as a basis for further investigations into complex non-stationary processes in the universe. These results are of scientific significance and offer new approaches applicable to the modeling and analysis of dynamic evolution of artificial and natural celestial bodies.

Approbation of the dissertation

1. Baisbayeva O.B. Ush osti beystatsionar denenің beystatsionar shardyң gravitatsiyalyқ órisindegi ilgerlemeli-aynalmaly қozғalysy. Materialy Mezhdunarodnoi nauchnoi konferentsii studentov i molodykh uchenykh "FARABI ÁLEMI", Kazakhstan, Almaty, KazNU imeni al-Farabi, 10.04.2018–12.04.2018, p. 86..

2. M. Minglibayev, A. Prokopenya. O. Baisbayeva. Evolution equations of translational rotational motion of a triaxial body with constant dynamical shape and variable size in a non-stationary central gravitational field. Proceedings The 7th International Congress of Serbian Society of Mechanics, Sremski Karlovci, Serbia, June 24-26, 2019, pp.129-130.

3. Mukhtar Minglibayev, Alexander Prokopenya, Oralkhan Baisbayeva. Analytical calculations of secular perturbations of translational-rotational motion of a non-stationary triaxial body in the central field of attraction. ACA 2019, 25th Conference on Applications of Computer Algebra, Montreal, Canada, July 16-20, 2019, pp. 156-157.

4. M.Dzh. Minglibaev, O.B. Baisbaeva. Postupatel’no-vrashchatel’noe dvizhenie nestatsionarnogo trekhosnogo tela v nestatsionarnom tsentral’nom pole tyagoteniya. XII Vserossiyskiy s’’ezd po fundamental’nym problemam teoreticheskoy i prikladnoy mekhaniki, August 19–29, 2019, Ufa, Republic of Bashkortostan, Russia. Proceedings in 4 volumes, *Vol. 1:* General and Applied Mechanics, pp. 708–710.

5. Mukhtar Minglibayev, Alexander Prokopenya, Oralkhan Baisbayeva. Secular Perturbations of Translational-Rotational Motion of a Non-stationary Triaxial Body in a Central Gravitational Field. CASTR 2019, Computer Algebra Systems in Teaching and Research, Volume VIII, Siedlce, Poland, September 25-29, 2019, pp.246-248.

6. M. Minglibayev, O. Baisbayeva. Translational-rotational motion of a triaxial body with variable compression in a non-stationary central gravity field. Computer Algebra Systems in Teaching and Research, Volume IX, Siedle University of Natural Sciences and Humanities, 2020, pp. 204-219.

Publications

Thomson Reuters or Scopus database:

1. Minglibayev M., Prokopenya A., Baisbayeva O. Evolution equations of translational-rotational motion of a non-stationary triaxial body in a central gravitational field. Theoretical and Applied Mechanics, V. 47 (2020), Issue 1, 63-80, DOI: https://doi.org/10.2298/TAM191130007M.
2. Prokopenya, A., Minglibayev, M., Baisbayeva,O. Analytical calculations of secular perturbations of translational-rotational motion of a non-stationary triaxial body in the central field of attraction. Lecture Notes in Computer Science (including subseries Lecture Notes in Artificial Intelligence and Lecture Notes in Bioinformatics), 2020, 12291 LNCS, 478–491, DOI:10.1007/978-3-030-60026-6_28.

Publications recommended by the Committee for Quality Assurance in Science and Higher Education of the Ministry of Science and Higher Education of the Republic of Kazakhstan:

1. M.Dzh. Minglibaev, O.B. Baisbaeva. K postupatel’no-vrashchatel’nomu dvizheniyu trekhosnogo nestatsionarnogo sputnika. Vestnik KazNPU, Seriya «Fiziko-matematicheskie nauki», Informatika 01(61), 2018, pp. 197–201.
2. M.Dzh. Minglibaev, O.B. Baisbaeva. Issledovanie postupatel’no-vrashchatel’nogo dvizheniya sputnika s peremennym tenzorom inercii v pole

prityazheniya sfericheskogo tela. Sovmestnyy vypusk. Vestnik VKGTU im. D. Serikbaeva – Vychislitel’nye tekhnologii Instituta vychislitel’nykh tekhnologiy Sibirskogo otdeleniya RAN, No. 3, Vol. 1, Part 3, 2018, pp. 179–195.

3. M.Dzh. Minglibaev, O.B. Baisbaeva. Postupatel’no-vrashchatel’noe dvizhenie trekhosnogo tela s peremennymi szhatiyami pri nalichii reaktivnykh sil i momentov. Khabarshy KazNPU im. Abaya, Seriya «Fiziko-matematicheskie nauki», No. 1(69), 2020, pp. 246–251.

The personal contribution of the author

The main results of the research presented in the dissertation work were obtained by the author independently. The author of the dissertation performed work on averaging and obtaining differential equations of secular perturbations and a review of the literature, analytical analysis of the equations of secular perturbations and conclusions, equations of secular perturbations were numerically solved and graphs were obtained, cases with constant and variable mass were compared, three- dimensional graphs of the first integral were constructed. The formulation of the problem and discussion of the results were carried out jointly with scientific consultants.

The scope and structure of the thesis

The dissertation consists of the introduction, three sections, the conclusion, the list of references and appendic.